Sources of water supply Ground water hydraulics

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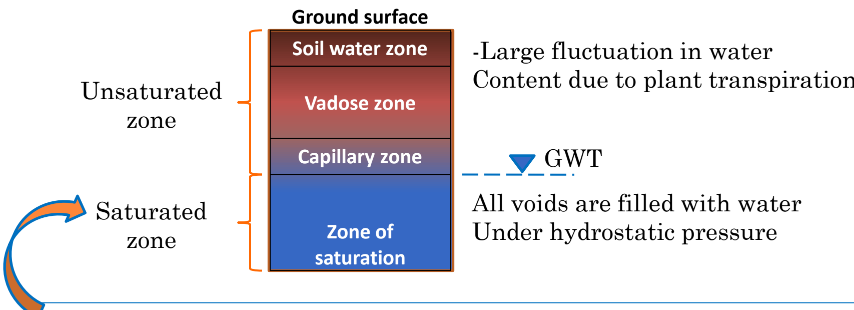
Groundwater

- Groundwater takes 0.62% of the total water in the hydrosphere
- 0.31% of the total water in the hydrosphere has depth less than 800m
- sand, gravel, and sandstones →good aquifers
- Limestone and shale that have caverns, fissures or faults can also be considered as good aquifers.
- Clay's ability to transmit water is very poor due to the very small particle sizes (< 0.0004 mm).

Subsurface Distribution of Water



Aquifer is a water-bearing formation that is saturated and that transmits large quantities of water.

Aquifer parameters

- *Porosity*: ratio of volume of voids to total volume

$$n = \frac{Vt - Vs}{Vt}$$

  - Vt is total volume of soil and Vs is the volume of solids
- *Specific yield(Sy)*: amount of water that will drain under the influence of gravity

$$Sy = \frac{Vd}{Vt}$$

- *Specific retention(Sr):* part that is retained as a film on rock surfaces and in very small openings.

$$Sr = \frac{Vr}{Vt} \qquad n = Sy + Sr$$

Aquifer parameters

- *Storage coefficient (S)*: the volume of water that an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head

$$S = \frac{Volume\ of\ water}{(Unit\ area)(unit\ head\ change)}$$

- *Hydraulic gradient (dh/dx)*: the slope of the piezometric surface or water table line in m/m. The magnitude of the head determines the pressure on the groundwater to move and its velocity.

Aquifer parameters

- *Hydraulic conductivity(K):* ratio of velocity to hydraulic gradient, indicating permeability of porous media.

$$K = \frac{QdL}{Adh}$$

- *Transmissivity:* the capacity of an aquifer to transmit water
  - measure of how easily water in a confined aquifer can flow through the porous media.

$T = Kb$, b = saturated thickness

Aquifer Types

- Unconfined and confined aquifers

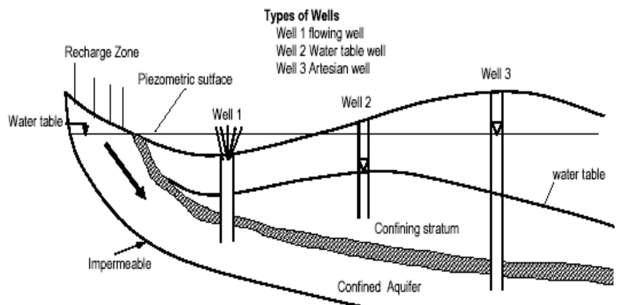


Fig. 1-4: Formation of

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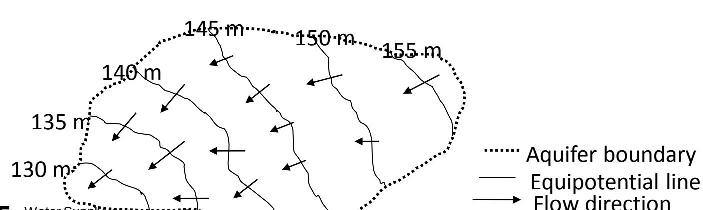
Groundwater flow

- Groundwater flows in the direction of decreasing head.
- **Equipotential lines**→ lines showing points having equal pressure.
- Flow direction is <u>perpendicular</u> to equipotential lines



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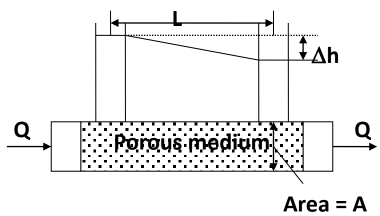
VELOCITY OF GW

- Velocity can be determined by Darcy's law ➔ $V = kS$
- **Darcy law** :Q through porous media is proportional to the head loss and inversely proportional to the length of the flow path.



$$V = \frac{Q}{A} = -K\frac{\Delta h}{L}$$

or

$$V = -K\frac{dh}{dL}\text{; for very small element}$$

- K = hydraulic conductivity and Δh is the head loss

DETERMINATION OF K

Laboratory methods

- ***Constant head permeameter:*** $K = \frac{VL}{Ath}$

V = volume water flowing in time t through of area A, length L, and with constant head h.

- ***Variable head permeameter :*** $K = \frac{r^2 L}{r_c^2 t}\ln\left(\frac{h_1}{h_2}\right)$

r = radius of the column in which the water level drops

r_c = radius of the sample

h_1, h_2 are heads at times t_1 and t_2, respectively

$t = t_2 - t_1$

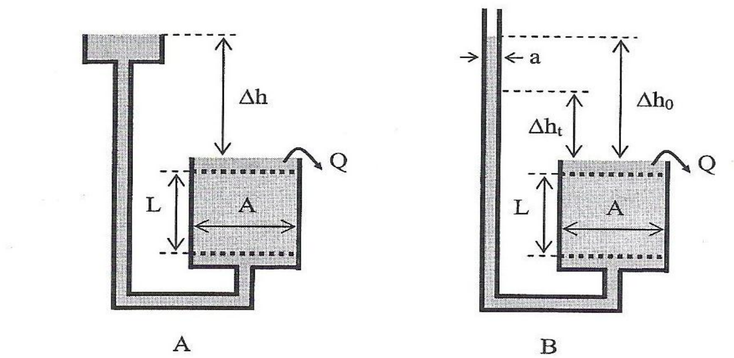


Fig. 1 Determination of the hydraulic conductivity with a permeameter: (a) constant head permeameter, and (b) falling head permeameter.

Determination of K…

Field Methods

- ***Pumping test***: constant removal of water from a single well and observations of water level declines at several adjacent wells.
- This is the most accurate way
- For anisotropic aquifers, the combined horizontal hydraulic conductivity:

$$K = \frac{\sum K_i Z_i}{\sum Z_i}$$

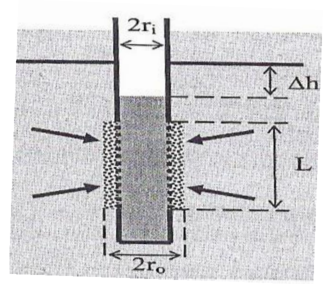
  - Where, K_i = K in layer i; Z_i = thickness of layer I

DETERMINATION OF K…

Field Methods…

- ***Slug test or piezometer test:*** the simplest method
- some volume of water is taken out from the piezometer and the subsequent rise of the water back to its original position is recorded in time.

$$K = \frac{r_i^2}{2Lt_o} \ln\left(\frac{L}{r_o}\right)$$



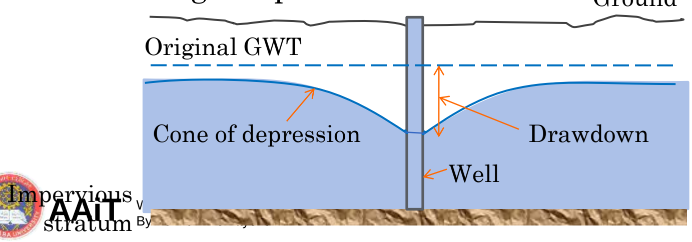
- r_i-inside radius,
- L- the length of the screen section,
- r_o-the outside radius
- t_o- characteristic time interval

HYDRAULICS OF WATER WELLS

- **Well:** hydraulic structure utilized to access water-bearing aquifers
  - Allows estimation of aquifer hydraulic properties
  - Provides direct access to ground water conditions
    1) Sampling
    2) Testing
    3) Resource Extraction
    4) Environmental Restoration

Hydraulics of water wells

- *Aquifer test*: studies involving analyzing the change, with time, in water levels in an aquifer caused by withdrawals through wells.
- *Drawdown/cone of depression*: is the difference between the water level at any time during the test and the original position.



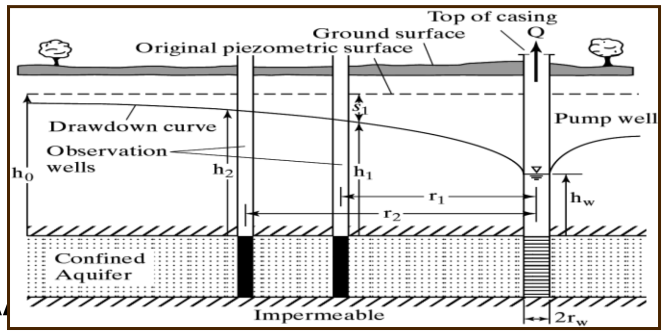
Steady state condition

- Cone of depression remains in equilibrium
- The water table is only slightly inclined
- Flow direction is horizontal
- Slopes of the water table and the hydraulic gradient are equal
- Aquifer: isotropic, homogeneous and infinite extent
- Well fully penetrating the aquifer

STEADY RADIAL FLOW TO A WELL-CONFINED

For horizontal flow, Q at any radius r equals, from Darcy's law,

$$Q = -2\Pi r b K \frac{dh}{dr} = -2\Pi r T \frac{dh}{dr}$$



STEADY RADIAL FLOW TO A WELL-CONFINED

- Integrating after separation of variables, with $h = h_w$ at $r = r_w$ at the well, yields Thiem Equation.

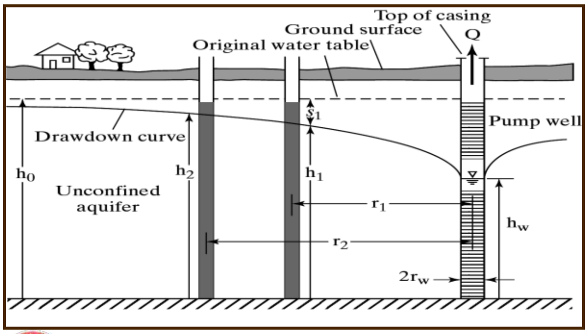
$$Q = 2\Pi T \frac{h - h_w}{\ln \frac{r}{r_w}}$$

- Near the well, transmissivity, T, may be estimated by observing heads h_1 and h_2 at two adjacent observation wells located at r_1 and r_2, respectively, from the pumping well.

$$T = Q \frac{\ln \frac{r_2}{r_1}}{2\Pi (h_2 - h_1)}$$

Steady Radial Flow to a Well-Unconfined

- radial flow in an unconfined, homogeneous, isotropic, and horizontal aquifer yields:



$$Q = -2\Pi r H K \frac{dH}{dr}$$

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Steady Radial Flow to a Well-Unconfined

- integrating, the flow rate in a unconfined aquifer from 2 to 1

$$Q = \Pi K \frac{(h_2{}^2 - h_1{}^2)}{\ln \frac{r_2}{r_1}}$$

- Solving for K, $K = \frac{Q}{\Pi (h_2{}^2 - h_1{}^2)} \ln \frac{r_2}{r_1}$

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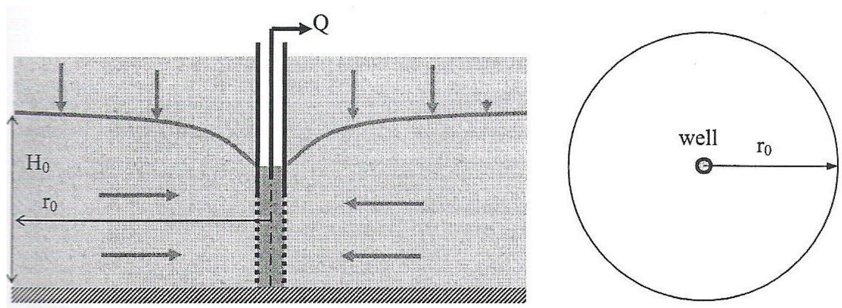
Radius of influence of steady state pumping wells



Example

- A 0.5 m well fully penetrates an unconfined aquifer of 30 m depth. Two observation well located 30 and 70 m from the pumped well have drawdowns of 7 m and 6.4 m, respectively. If the flow is steady and K = 74 m/d
  - what would be the discharge
  - Estimate the drawdown at the well

SOLUTION

- For unconfined well Q is given as

$$Q = \Pi K \frac{(h_2^{\ 2} - h_1^{\ 2})}{\ln \frac{r_2}{r_1}}$$

h_1= 30 -7 = 23 m, and h_2 = 30 – 6.4 = 23.6 m

r_1 = 30 m and r_2 = 70 m

$$Q = \Pi \times 74 \frac{(23.6^2 - 23^2)}{\ln \frac{70}{30}} = 7671.54\, m^3 / day$$

SOLUTION

- Drawdown at the well,
  - using the h_1=23 m and r_w=0.5m/2=.25 m, we have hw

$$Q = \Pi K \frac{(h_1^{\ 2} - h_w^{\ 2})}{\ln \frac{r_1}{r_w}} = \Pi \times 74 \frac{(23^2 - h_w^{\ 2})}{\ln \frac{30}{0.25}} = 7671.54\, m^3 / day$$

Solving for hw, we have h_w = 19.26 m

So the drawdown would be 30.0 – 19.26 = 10.74 m

EXAMPLE

- Design a tube well for the following data
  - Yield required = 0.1 m^3/sec
  - Thickness of confined aquifer = 25 m
  - Radius of confined aquifer = 250 m
  - Permeability coefficient = 70 m/day
  - Drawdown at the well = 6 m

SOLUTION

- For confined aquifer Q is given by $Q = 2\Pi bK \frac{h - h_w}{\ln \frac{r}{r_w}}$
- Taking between the well and at the radius of influence(R) we have
  - $h-h_w$ = 6 m
  - b = 25 m
  - R = 250 m

$$Q = 0.1m^3/\sec = 2\Pi \times 25m \times \left(\frac{70\, m/day}{86400 \sec/day}\right) \frac{6m}{\ln \frac{250}{r_w}}$$

Solving for r_w, we get r_w = 0.12 m or 12 cm
Thus, diameter of the well is 24 cm or 25 cm

Transient or unsteady state condition

Assumptions:

- The aquifer is homogenous, isotropic, uniformly thick, and of infinite areal extent
- Prior to pumping the piezometric surface is horizontal
- The fully penetrating well is pumped at constant rate
- Flow is horizontal within the aquifer
- Storage within the well can be neglected
- Water removed from storage responds instantaneously with a declining head

Transient or unsteady state condition

- The governing equation in plane polar coordinates is:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Where, h = head

r = radial distance

S = storage coefficient

T = transmissivity

- Solution methods to solve the governing equation: Theis and Cooper-Jacob methods

Theis Method

Theis assumed the following :

- T is constant during the test to the limits of the cone of depression
- The water withdrawn from the aquifer is entirely from the storage and discharged with the decline in head.
- The discharging well penetrates the entire thickness of the aquifer.
- the diameter of the well is small relative to the pumping rate so that the storage in the well is negligible.

Theis Method...

- Theis solution is written as:

$$s' = \frac{Q}{4\Pi T}\int_u^\infty \frac{e^{-u}}{u}du$$

- The integral in the Theis equation is written as *W(u)*:

W(u) = – 0.5772 – ln(u) + u – u2/2·2! + u3/3·3! – u4/4·4! + ...

- Therefore:

$$s' = \frac{Q}{4\Pi T}W(u) \qquad u = \frac{r^2 S}{4Tt}$$

s' = drawdown, *W(u)* = well function, *Q* = discharge at the well, S = storage coefficient, T = transmissivity, t = time

THEIS METHOD…

- **Procedure:**
  - Plot the type curve: W(u) vs. u or 1/u and on a log-log paper
  - Plot the observed data: s' vs. r^2/t or t/r^2 on a transparent log-log paper
  - Superimpose the observed plot on the type curve
  - Adjust the observed plot in such a way that most of the points lie on the type curve.
  - **Select one matching point and take the corresponding readings for W(u), u, s' and r^2/t.**
  - Compute T from the Theis equation: $T = \frac{Q}{4\Pi s'} W(u)$
  - Determine S from the equation for u: $S = 4Tu \frac{1}{r^2/t}$

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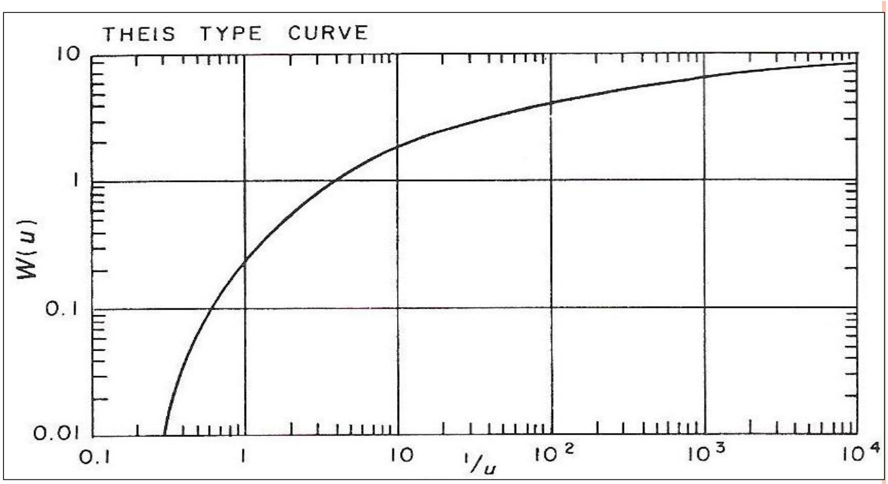

W(u) = – 0.5772 – ln(u) + u – u2/2·2! + u3/3·3! – u4/4·4! + …

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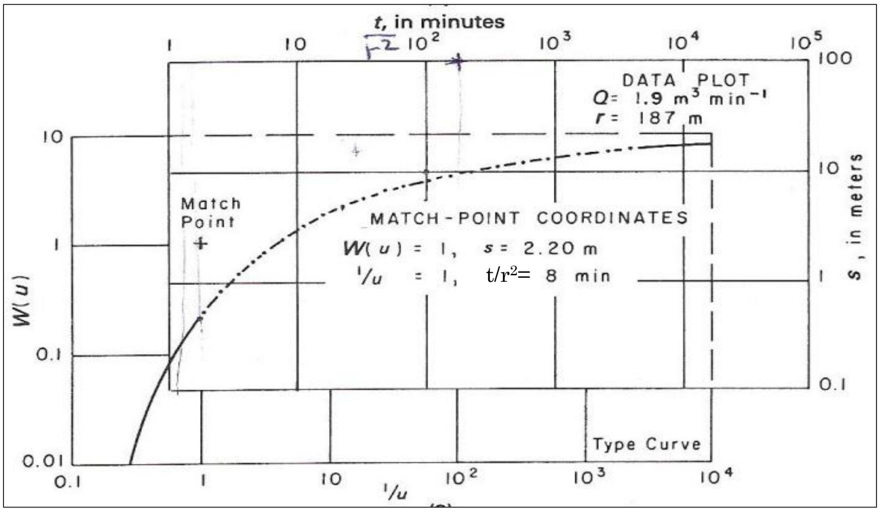


THEIS METHOD…

- For a known S and T, we can compute ***s'*** directly at a given ***r*** from the well as a function of time:
  - First compute $u = r^2S / (4Tt)$
  - Then, calculate for W(u)
  - Finally, $s' = \frac{Q}{4\Pi T} W(u)$

Cooper-Jacob Method

- Theis equation applies to all times and places if the assumptions are met but Jacob's method applies only under certain additional equations.
- Facts:
  - At the start of withdrawals, the entire cone of depression has unsteady shape
  - After some time, the cone of depression begins to have a relatively steady shape
- The *Jacob method* is applicable only to the *zone in which steady shape* conditions prevail or to the *entire cone only after steady conditions* have developed

Cooper-Jacob Method…

- Cooper and Jacob noted that for small values of ***r*** and large values of ***t***, the parameter ***u = r²S/4Tt*** becomes very small so that the infinite series can be approximated by:
  - ***W(u) = – 0.5772 – ln(u)*** *(neglecting higher terms)*

$$s' = \frac{Q}{4\Pi T}\left(-0.5772 - \ln u\right)$$

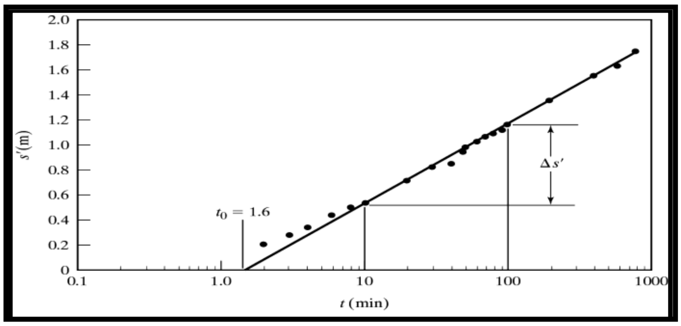
  - Rearranging the above equation

$$s' = \frac{2.3Q}{4\Pi T}\log\left(\frac{2.25Tt}{r^2 S}\right)$$

Cooper-Jacob Method…

- plot of s' vs. log (t), projection of the line back to s' = 0, where t = t_0 yields the following relation:

$$0 = \frac{2.3Q}{4\Pi T}\log\left(\frac{2.25Tt_0}{r^2 S}\right) \rightarrow S = \frac{2.25Tt_0}{r^2}$$



Cooper-Jacob Method…

- Replacing *s* by *Δs*, where *Δs* is the drawdown difference per unit log cycle of *t*:

$$T = \frac{2.3Q}{4\Pi\Delta s'}$$

- The Cooper-Jacob method first solves for *T* and then for *S* and is only applicable for small values of u (***u*** **< 0.01).**

Example

A fully penetrating artesian well is pumped at a rate Q = 1600 m^3/d from an aquifer whose S and T values are 4 x 10^{-4} and 0.145 m^2/min, respectively.

- What is the drawdown at a distance of 100 m after a) 1 hr and b)1 day of pumping?
- Estimate the radius of influence after 1 h and 1 day of pumping

Solution

- For t = 1 hr = 60*60 = 3600 sec
- First calculate u
- u = r^2S / (4Tt) = 100^2 x 4 x 10^{-4}/ (4 x 0.145/60 x 3600)= 0.1149
- Read/ calculate W(u)=1.698
- Thus, the drawdown becomes
- s= 1600/86400*1.698/(4 x π x 0.145/60) = 1.035 m

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SOLUTION….

- For t = 1 day = 86400 sec
- First calculate u
- u = r^2S / (4Tt) = 100^2 x 4 x 10^{-4}/ (4 x 0.145/60 x 86400)= 0.00479
- Read/ calculate W(u)=4.769
- Thus, the drawdown becomes
- s = 1600/86400*4.769/(4 x π x 0.145/60) = 2.908 m

SOLUTION

- Determine the radius of influence
- We may use Jacob's formula to determine the radius of influence

$$S = \frac{2.25Tt_0}{r^2}$$

For time t=1hr= 3600 sec

$$\mathrm{r}^2 = \frac{2.25\mathrm{Tt_o}}{\mathrm{S}} = \frac{2.25 \times 0.145/60 \times 3600}{4 \times 10^{-4}} = 48937.5m^2$$

Thus, the radius of influence, r = 221.21 m

Solution

For time t=1day = 86400 sec

$$r^2 = \frac{2.25Tt_o}{S} = \frac{2.25 \times 0.145/60 \times 86400}{4 \times 10^{-4}} = 1174500 m^2$$

Thus, the radius of influence, r = 1083.74 m

Well Hydraulics

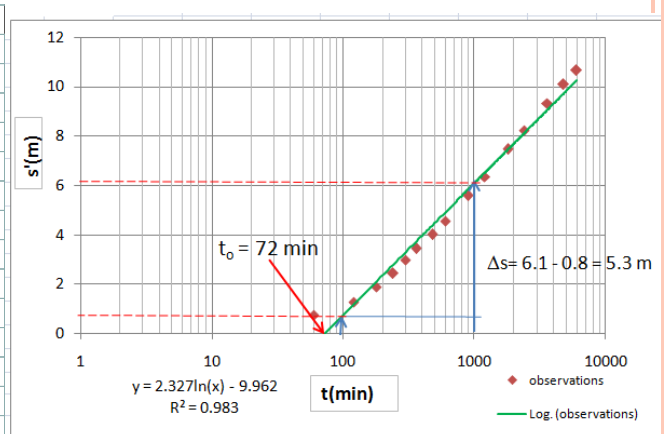
A well is pumped at a rate of 0.75 m^3/min. at an observation well 30 m away, the drawdowns were noted as a function of time as shown below:

t, days	s, m		t, days	s, m		t, days	s, m
1	0.75		6	3.45		30	7.47
2	1.3		8	4.02		40	8.24
3	1.9		10	4.57		60	9.34
4	2.45		15	5.6		80	10.1
5	3		20	6.37		100	10.66

- Determine the values of T and S using Cooper-Jacob's method.

Solution

t, days	t, min	s, m
1	60	0.75
2	120	1.3
3	180	1.9
4	240	2.45
5	300	3
6	360	3.45
8	480	4.02
10	600	4.57
15	900	5.6
20	1200	6.37
30	1800	7.47
40	2400	8.24
60	3600	9.34
80	4800	10.1
100	6000	10.66



Solution…

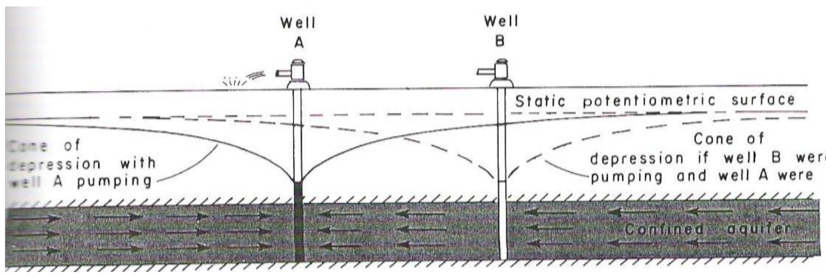
- From the graph we have to = 72 min and Δs = 5.3 m
- And Q = 0.75 m³/min and r = 30 m
- Thus,

$$T = \frac{2.3Q}{4\Pi \Delta s'} = \frac{2.3 \times 0.75}{4\pi \times 5.3} = 0.0259\, m^2 / \min$$

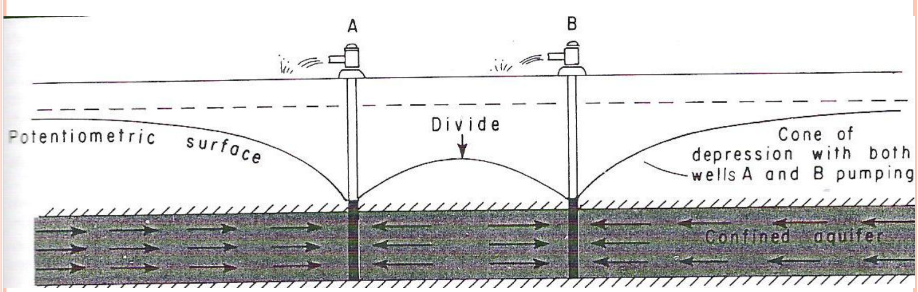
$$S = \frac{2.25 T t_0}{r^2} = \frac{2.25 \times 0.0259 \times 72}{30^2} = 0.00466$$

INTERFERENCE OF WELLS

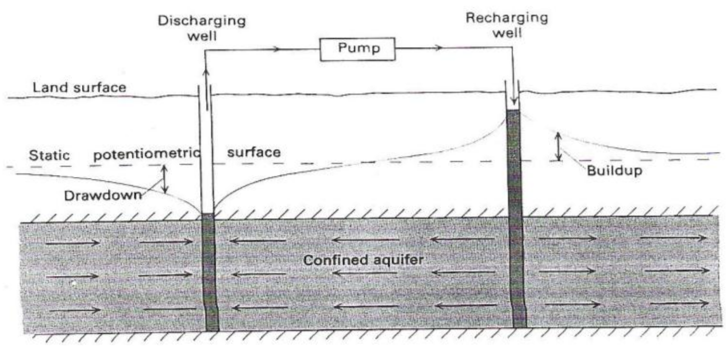
- The combined drawdown at a point is equal to the sum of the drawdowns caused by individual wells.
- Reduced yield for each of the wells.



Resultant drawdown



PUMPING AND RECHARGING WELLS



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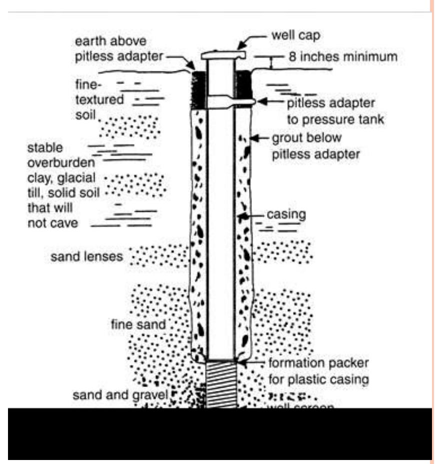
WELL CONSTRUCTION

- Well construction depends on
  - the flow rate,
  - depth to groundwater,
  - geologic condition,
  - casing material, and
  - economic factors
- Shallow and deep well construction



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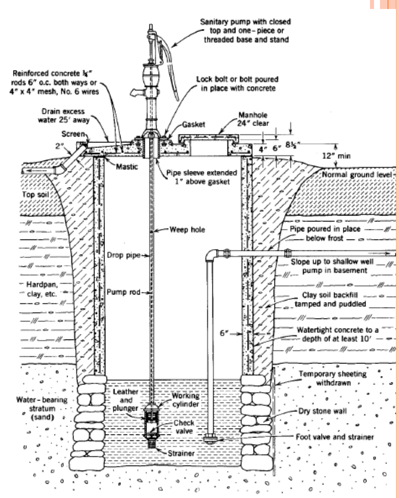
SHALLOW WELL CONSTRUCTION

- Shallow wells are less than 30 m deep
- constructed by
  - digging,
  - boring,
  - driving, or
  - jetting methods.

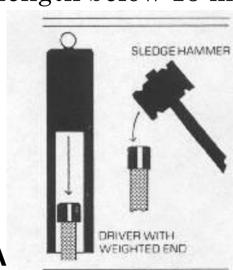
SHALLOW WELL CONSTRUCTION

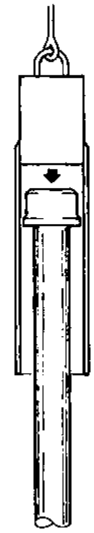
- *Dug wells:* excavated by hand and are vertical wells.
  - diameter > 0.5 m and depth < 15 m.
  - Lining and casing :concrete or brick.



SHALLOW WELL CONSTRUCTION

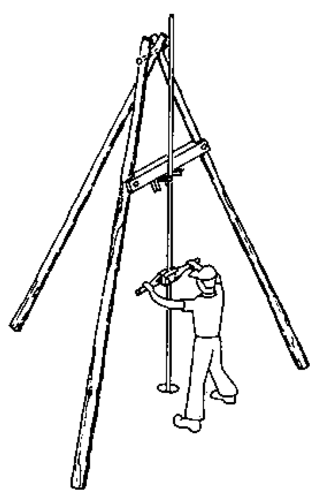
- *Driven wells:* a series of pipe lengths driven vertically downward by repeated impacts into the ground.
  - diameters 25 – 75 mm
  - Length below 15 m.





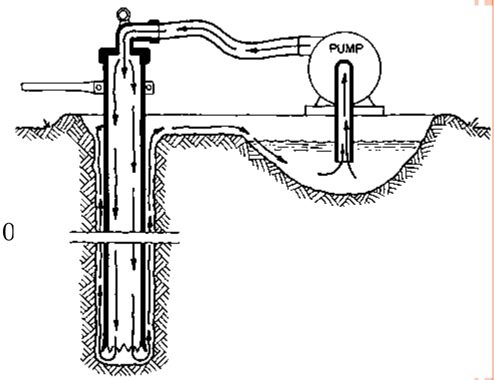
SHALLOW WELL CONSTRUCTION

- *Bored wells*: constructed with hand-operated or power-driven augers.
  - Diameters of 25 to 900 mm
  - depths up to 30 m



SHALLOW WELL CONSTRUCTION

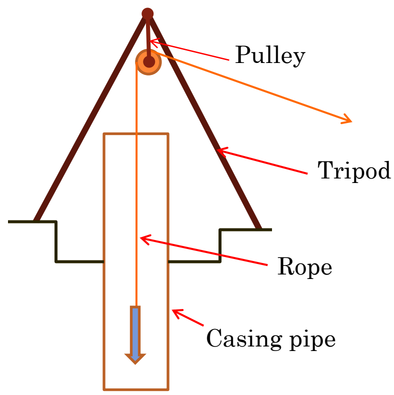
- *Jetted wells:* a high-velocity stream of water directed vertically downward, while the casing that is lowered into the hole conducts the water and cuttings to the surface.
  - Small-diameter holes, up to 10 cm,
  - depths up to 15 m
  - useful for observation wells and well-point systems for dewatering purposes.



DEEP WELL CONSTRUCTION

- Deep wells constructed by *percussion (cable tool) drilling* or *rotary drilling methods.*
- Percussion drilling: regular lifting and dropping of a string of tools, with a sharp bit on the lower end to break rock by impact.
  - for consolidated rock materials to depths of 600 m.

PERCUSSION DRILLING



DEEP WELL CONSTRUCTION

- Rotary method: consists of drilling with a hollow, rotating bit, with drilling mud or water used to increase efficiency. No casing is required with drilling mud because the mud forms a clay lining on the wall of the well. Drilling mud consists of a suspension of water, bentonite clay, and various organic additives.
  - A rapid method for drilling in unconsolidated formations
  - Air rotary methods use compressed air in place of drilling mud and are convenient for consolidated formations.
  - Drilling depths can exceed 150 m

HYDRAULIC ROTARY DRILLING